Sec. 3.2 Vertex of a Parabola

Solving on the TI Calculator:
**Use this when it is not possible to get an exact answer from solving algebraically. Always round to two decimal places.

1. ZERO function: Use this function when the equation is set equal to zero.
2. INTERSECT function: Use this function when the equation is not set equal to zero.

Steps for Using Zero Function
1. Write the equation in the form (expression in x) = 0.
2. Graph Y1 = (expression of x).
3. Use the ZERO or ROOT function to determine each x-intercept of the graph.
4. Choose an appropriate window to see entire graph.

Steps for Using Intersect Function
1. Graph Y1 = (expression on left side of equation).
2. Graph Y2 = (expression on right side of equation).
3. Use INTERSECT to determine the x-coordinate of each point of intersection.

Ex. Find the solutions to $x^2 + 2x - 4 = 0$.

$x = -3.24$ $x = 1.24$

Ex. Find the solutions to $2x^3 - 3x + 1 = 0$.

$x = -1.37$ $x = .37$ $x = 1$

Ex. Find the solutions to $3x^4 + 2 = 2x + 5$.

$3x^4 - 2x - 3 = 0$ $x = -.82$ $x = 1.15$

Solving by Hand
Quadratic Equations: form of $ax^2 + bx + c = 0$.

a. To solve, first write equation in standard form or factored form.
b. If equation is in standard form, try to factor it.
c. Set each factor to 0 and solve individually.
d. Solution set of answers written as { }.
e. Ex. Solve $x^2 = 12 - x$.

$x^2 + x - 12 = 0$
$(x+4)(x-3) = 0$
$x + 4 = 0$ $x - 3 = 0$
$x = -4$ $x = 3$

*****If you get the same answer for both factors, it is called having a root of multiplicity 2.**

Quadratic Equations: form of $ax^2 + bx + c = 0$ that do not factor, use the quadratic formula.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ex. Find the real solutions of $3x^2 - 5x = -1$.

$3x^2 - 5x + 1 = 0$

$\frac{5 \pm \sqrt{(-5)^2 - 4(3)(1)}}{2(3)}$

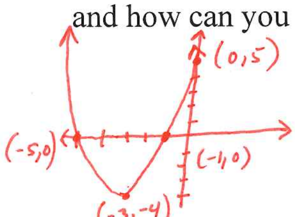
$\frac{5 \pm \sqrt{25 - 12}}{6}$

$\frac{5 \pm \sqrt{13}}{6}$

$x = 1.43 \quad x = .23$

Quadratic Functions in the form of $f(x) = (x + b)^2 - c$ are called vertex form and you can use this to graph.

Ex. Sketch the graph of $f(x) = (x + 3)^2 - 4$ and find the vertex. What are the intercepts and how can you use the formula to find the minimum or maximum of the graph?



$f(0) = (0+3)^2 - 4$
$= 9 - 4$
$= 5$

MINIMUM WILL BE Y COORDINATE OF VERTEX IF OPENS UP, MAX IF OPENS DOWN

$0 = (x+3)^2 - 4$
$\sqrt{4} = \sqrt{(x+3)^2}$

$2 = x + 3$
$-1 = x$

$-2 = x + 3$
$-5 = x$

Ex. Put the following quadratic equations into vertex form by completing the square.

a. $s(x) = x^2 - 6x + 8$

$(x^2 - 6x + \quad) + 8$
$(x^2 - 6x + 9) - 9 + 8$
$s(x) = (x - 3)^2 - 1$

b. $t(x) = -4x^2 - 12x - 8$

$= -4(x^2 + 3x \quad) - 8$
$= -4(x^2 + 3x + \frac{9}{4}) - 9 - 8$
$t(x) = -4(x + \frac{3}{2})^2 - 17$

Ex. Given the point (-3, 2) is the vertex and the point (0, 5) is on the graph, find an equation to model this situation.

$y = a(x+3)^2 + 2$
$5 = a(0+3)^2 + 2$
$5 = 9a + 2$
$3 = 9a$
$\frac{1}{3} = a$

$y = \frac{1}{3}(x+3)^2 + 2$

Ex. David has available 400 yards of fencing and wishes to enclose a rectangular area.

a. Express the area A of the rectangle as a function of x, where x is the length of the rectangle. $A = x(200 - x)$
$= 200x - x^2$

length $= x$
width $= \frac{400 - 2x}{2} = 200 - x$

b. For what value of x is the area the largest? VERTEX: $\frac{-b}{2a} \quad \frac{-200}{2(-1)} = \frac{-200}{-2} = \quad x = 100$

c. What is the maximum area? $A = 200(100) - 100^2$
$= 20000 - 10000$
$A = 10{,}000 \text{ yd}^2$

Summary:

Option 1:

a. Complete the square in x to write the equation in the form $f(x) = a(x - h)^2 + k$.
b. Graph the function in stages using transformations.

Option 2:

a. Determine the vertex using your equation.
b. Determine the axis of symmetry.
c. Determine the y-intercept, f(0).
d. Use your discriminant and the quadratic formula to find x-intercepts.
e. Determine an additional point by using the y-intercept and the axis of symmetry.
f. Plot the points and draw the graph.

Ex. Determine the equation of the graph whose vertex is (-3, 5) and whose y-intercept is –4. Then find the graphs maximum or minimum value.

$y = a(x+3)^2 + 5$

$-4 = a(0+3)^2 + 5$

$-4 = 9a + 5$

$-9 = 9a$

$-1 = a$

$y = \ (x+3)^2 + 5$

minimum value at vertex : (-3,5)

HW: pg 115- 117, # 2, 3, 5, 7, 9, 10, 13, 16, 19, 22, 23, 25, 29, 30, 31